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# Earth Conductivity Estimation from Through-the-Earth Measurements of 94 Coal Mines Using Different Electromagnetic Models

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#### Abstract

Through-the-Earth (TTE) communication systems require minimal infrastructure to operate. Hence, they are assumed to be more survivable and more conventional than other underground mine communications systems. This survivability is a major advantage for TTE systems. In 2006, Congress passed the Mine Improvement and New Emergency Response Act (MINER Act), which requires all underground coal mines to install wireless communications systems. The intent behind this mandate is for trapped miners to be able to communicate with surface personnel after a major accident-hence, the interest in TTE communications. To determine the likelihood of establishing a TTE communication link, it would be ideal to be able to predict the apparent conductivity of the overburden above underground mines. In this paper, all 94 mine TTE measurement data collected by Bureau of Mines in the 1970s and early 1980s, are analyzed for the first time to determine the apparent conductivity of the overburden based on three different models: a homogenous half-space model, a thin sheet model, and an attenuation factor or Q-factor model. A statistical formula is proposed to estimate the apparent earth conductivity for a specific mine based on the TTE modeling results given the mine depth and signal frequency.

#### **Index Terms**

Communication; conductivity; electromagnetic field; Extremely Low Frequency (ELF); Throughthe-Earth (TTE); Very Low Frequency (VLF)

#### I. Introduction

In coal mines, frequency and effective electrical conductivity of the overburden are factors that determine the maximum range through which a TTE signal can successfully propagate. The effective electrical conductivity of the overburden cannot be controlled and depends on the mine geological properties and varies between different mine sites. Ideally, we would like to be able to predict the apparent conductivity of the overburden to determine the likelihood of being able to establish a TTE communication link. This includes communications between locations within the mine (horizontal communication) and between the underground and the surface (vertical communication). Obtaining information on the overburden electrical properties is useful for evaluating and improving the performance and reliability of a TTE system at a given mine. The limited information on the

electrical characteristics of overburden above U.S. coal mines, however, prevents the development of a detailed theoretical approach.

In the 1970s and early 1980s, the Bureau of Mines measured the propagation of TTE signals for frequencies ranging from 600 Hz to 3000 Hz for 94 representative mines distributed throughout the United States. The TTE transmission data collected at the 27 coal mines were initially analyzed to estimate the apparent earth conductivity based upon a homogeneous half-earth model [1]. This model was used to predict the apparent earth conductivity, but it also led to unrealistic results of the conductivity decreasing with increasing frequency and depth. Hill and Wait then proposed a thin sheet model to simulate conducting materials, such as pipes, cables, metal sheets, or higher conductivity layers near the surface [2]. This model provides an explanation for the depth dependency of the conductivity. It also predicts the magnitude of decrease in apparent conductivity with frequency by appropriately setting the properties of the conducting sheet. In this paper, the TTE data from all 94 mines are analyzed to determine the apparent conductivity of the overburden based on homogenous half-space model and thin sheet model. Alternately, a mathematically simple model, the Qfactor model, combining features of the two models mentioned above is proposed to predict the dependency of apparent conductivity on both frequency and depth. There is agreement on this prediction based on the thin sheet model and Q-factor model by appropriately choosing the properties of the highly conducting thin sheet layer. Based on the TTE modeling results, we develop statistical formulas that can be used to estimate the earth conductivity for a specific mine, given the mine depth and signal frequency.

# II. Background

A number of techniques are available for probing the earth electromagnetically with a transmitter and receiver, where conductivity information is contained in the received signal [3-6]. In the mid-1970s, Lagace et al., under the direction of the Bureau of Mines (BOM), conducted extensive TTE propagation measurements yielding the magnetic field (H-field) strengths at 94 coal mine sites [7]. The 94 mines were well-distributed over U.S. coal fields and were selected from the total mine population based on the depth and the number of mine workers (Table 1). In parallel, the BOM conducted additional TTE measurements at 27 coal mines which were selected from the 94 mines [1]. The sampling procedure used to select the 94 mines out of all US coal mines was based on the following principles:

- 1. each mine had a chance of being selected for this test;
- **2.** the probability of selection was known beforehand and was based on the relative size of the mine in terms of the number of miners employed;
- **3.** the selection process was random;
- 4. all depth intervals were selected;
- 5. test results could be used to make valid inferences about all mines [7].

# III. Methodology

#### A. Homogenous half-space model

The homogenous half-space model can be illustrated by setting either the conducting sheet depth to d=0 or  $\sigma=\sigma_0$ , in Fig. 1. The vertical magnetic dipole source (small horizontal loop) has a magnetic moment IA and is located at z=-h on the z axis of a cylindrical coordinate system  $(\rho, \varphi, z)$ , where I is the current through the loop wire and A is the area formed by the circular loop. The earth media has a conductivity of  $\sigma_0$  and an intrinsic propagation constant

of  $\gamma_0 = \sqrt{\mu\omega(j\sigma_0 - \omega\varepsilon_e)}$ , where j is the square root of -1,  $\omega$  the operating angular frequency,  $\mu$  the permeability of the earth or air, and  $\varepsilon_e$  the earth dielectric constant. The displacement currents are usually very small and will be neglected for all TTE frequencies compared to the conduction currents, so approximation  $\gamma_0 \approx \sqrt{j\omega\mu\sigma_0}$  is reasonable. The vertical H-field at the surface can be derived from a magnetic Hertz vector with only a z component, along with the application of appropriate boundary conditions [8]. It can also be obtained by setting the conductivities  $\sigma_1$  and  $\sigma_2$  both equal to  $\sigma_3$ , or the thickness  $\sigma_3$  of the upper layer can be considered to vanish and  $\sigma_2 = \sigma$  in a 2-layer model [8]. This gives:

$$H_z = b_0 Q_{homo}(\omega, \sigma_0, h), \quad (1)$$

where

$$Q_{homo}(\omega, \sigma_0, h) = \frac{h^3}{2} \int_0^\infty \lambda^2 T(\lambda) e^{-\lambda z} J_0(\lambda \rho) d\lambda, \quad (2)$$

and

$$T(\lambda) = \frac{2\lambda}{k_0 + \lambda} e^{-k_0 h}.$$
 (3)

In the equations above,  $b_0$ =IA/ $(2\pi h^3)$  is the value of H-field on the axis a distance h above a loop in free space,  $Q_{homo}(\omega, \sigma_0, h)$  represents the attenuation factor due to the conductive earth.  $J_0$  is the first order of Bessel function of the first kind, and  $k_0 = (\lambda^2 + \gamma_0^2)^{1/2}$ .

### B. Thin sheet model

As we will see later, one result from applying the homogeneous model to the data of the 94 mines is that the apparent conductivity appears to decrease as the mine depth increases, which is contradictory to the model itself. However, this dependency might be explained by the presence of a thin highly conducting layer at the surface of the earth. In the thin sheet model, as depicted in Fig. 1, the highly conducting thin sheet represents the overall effect of possible surface metal structures, such as cables, pipes, cased bore holes, etc., as well as the relatively high conductivity at the surface which usually contains more dissolved salt and mineral substances. Note that  $d \ll h$ . Wait and Spies have derived the H-field from the electric vector potential [8]. Following their work, here we use magnetic Hertzian potential to obtain the vertical H-field in the air based on potential theory.

For the earth layer (z<0) and free space (z>0), the magnetic Hertzian potential  $\Pi^*$  satisfies the wave equation except at the exciting source:

$$\left\{ \begin{array}{ccc} (\nabla^2 - \gamma_0{}^2)\Pi_0^*{=}0, & z{<}0 \\ \nabla^2\Pi_1^*{=}0, & z{>}0 \end{array} \right., \quad {}^{(4)}$$

where

$$\gamma_0^2 = \mu \omega (j\sigma_0 - \omega \varepsilon_0) \approx j\mu \omega \sigma_0,$$
 (5)

is the intrinsic propagation constant. For free space (z>0),  $\sigma_1$  = 0, hence,  $\gamma_1 \approx 0$ .

In a cylindrical coordinate system, the fields in the half-space (z<0) can be expressed in terms of Hertzian potential  $\Pi_{0}^{*}$  in which 0 denotes the medium 0 (earth):

$$\begin{cases}
\{H_{0\rho}, H_{0z}\} = \left\{ \frac{\partial^2 \Pi_0^*}{\partial \rho \partial z}, \left( -j\mu\omega\sigma_0 + \frac{\partial^2}{\partial z^2} \right) \Pi_0^* \right\} \\
E_{0\varnothing} = j\mu\omega \frac{\partial \Pi_0^*}{\partial \rho},
\end{cases} (6)$$

and the fields in the free space (z>0) can be expressed in terms of  $\Pi_1^*$ :

$$\begin{cases}
\{H_{1\rho}, H_{1z}\} = \left\{\frac{\partial^2 \Pi_1^*}{\partial \rho \partial z}, \frac{\partial^2}{\partial z^2} \Pi_1^*\right\} \\
E_{1\varnothing} = j\mu \omega \frac{\partial \Pi_1^*}{\partial \rho}.
\end{cases} (7)$$

Note that the magnetic Hertzian potential  $\Pi^*$  has only a z component; i.e.,  $\Pi^* = \Pi^* \hat{z}$ . The magnetic Hertzian potential in each region is listed below:

$$\Pi_0^* = \frac{IA}{4\pi} \int_0^\infty J_0(\lambda \rho) \left( \frac{\lambda}{k_0} e^{-k_0|z+h|} + R_0(\lambda) e^{k_0 z} \right) d\lambda, z < 0, \quad (8)$$

$$\Pi_1^* = \frac{IA}{4\pi} \int_0^\infty T_1(\lambda) e^{-k_1 z} J_0(\lambda \rho) d\lambda, z > 0. \quad (9)$$

 $R_0(\lambda)$  and  $T_1(\lambda)$  are unknowns and can be determined by the application of the boundary conditions. The boundary condition here requires that at the layer interface the azimuthal Efield is continuous and the tangential H-field is discontinuous by the amount of longitudinal current per unit length carried by the thin sheet:

$$R_0(\lambda) = \frac{\lambda(k_0 - k_1 - j\omega\mu\sigma d)}{k_0(k_0 + k_1 + j\omega\mu\sigma d)} e^{-hk_0}, \quad (10)$$

$$T_1(\lambda) = \frac{2\lambda}{k_0 + k_1 + j\omega\mu\sigma d} e^{-h\mathbf{k}_0}. \quad (11)$$

The surface vertical H-field in (7) then is given by:

$$H_z = b_0 Q_{thin}(\omega, \sigma_0, h, \sigma, d), \quad (12)$$

in which

$$Q_{thin}(\omega, \sigma_0, h, \sigma, d) = \int_0^\infty \frac{\lambda h^3 k_1^2}{k_0 + k_1 + i\omega\mu\sigma d} e^{-k_1 z - k_0 h} J_0(\lambda \rho) d\lambda. \quad (13)$$

For free space,  $k_1 = (\lambda^2 + \gamma_1^2)^{1/2} = \lambda$ . Equation (13) then can be rewritten as:

$$Q_{thin}(\omega, \sigma_0, h, \sigma, d) = \int_0^\infty \frac{\lambda^3 h^3}{k_0 + \lambda + j\omega\mu\sigma d} e^{-\lambda z - k_0 h} J_0(\lambda \rho) d\lambda. \quad (14)$$

To avoid exponential attenuation when the signal passes through the highly conducting thin sheet, the value of  $\gamma_0 d$  in this model is required to be small enough ( $\gamma_0 d < 1$ ). Similarly as in (1),  $Q_{thin}(\omega, \sigma_0, h, \sigma, d)$  here represents the attenuation factor due to the conductive earth and the thin sheet. An interesting feature of the attenuation factor  $Q_{thin}(\omega, \sigma_0, h, \sigma, d)$  in (14), is that the dependence on  $\sigma d$  is algebraic rather than exponential. This type of algebraic dependence is typical of thin conducting sheets regardless of the geometry [8].

#### C. Q-factor model

Since Q in (1) and (12) monotonically decreases with  $\sigma_0$  and/or  $\sigma$ , an apparent conductivity value  $\sigma_a$  can be determined by assuming reasonable input values for the homogeneous half-space and thin sheet model. The procedure is to compute  $Q_{thin}(\omega, \sigma_0, h, \sigma, d)$  using reasonable values of  $\sigma_0$  and  $\sigma d$ , and then equate the magnitude of  $Q_{thin}$  to that of a homogeneous half-space and determine the value of  $\sigma_a$ :

$$Q_{homo}(\omega, \sigma_a, h) = Q_{thin}(\omega, \sigma_0, h, \sigma, d).$$
 (15)

#### IV. Numerical Evaluation and Results

The numerical integration of (2) and (14) can be evaluated by using a variable exchange, as shown below [9]:

$$x = \lambda h; D = \rho/h; Z = z/h; T = h/\delta; \text{ with } \delta = \sqrt{\frac{2}{\omega\mu\sigma}}.$$
 (16)

Then the wave number  $k_i$ , can be rewritten as  $k_0 = x/h$ ;  $k_1 = \frac{1}{h} \sqrt{x^2 + jH^2}$ , with  $H = 2h/\delta$ .

#### A. Based on the homogenous half-space model

For the TTE tests at the 94 mines, four transmission frequencies-630, 1050, 1950, and 3030 Hz-were used at each site. The magnetic moment, M=NIA (N is the number of turns of wire), for the in-mine transmitting loop was recorded and calibrated. Corrections have also been made on the overburden depth h for all the mine sites to account for possible horizontal offsets from the point directly above the transmitters [7]. After normalizing the surface vertical H-field to the corresponding M, the apparent earth conductivity,  $\sigma_a$ , can then be obtained by solving (1).

The resulting apparent earth conductivity distribution with overburden depth interval is listed in Table 2. The computed conductivity values for the mine with the least overburden depth appear to be a large outlier compared to the rest of the data and were excluded from further consideration. Also, because of the large expected uncertainty in the conductivity estimates for large Q, the data for about 25% of the 94 mines in which |Q|>0.5 were excluded. By examining Table 2, we can see that the estimated conductivity values tend to decrease with increasing depth by a factor of 25-30 over the depth range at each given frequency. This number is in contrast to the factor of 10 obtained by Durkin based on the data of 27 mines [10]. The estimated apparent conductivity in Table 2 also shows a dependence on frequency that decreases by a factor of  $\sim$ 3 over the frequency range at each given depth interval. Analyzing only 27 out of 94 mine data in [10], does not quite predict the magnitude of decrease in the apparent conductivity with depth and frequency. However, we would not expect the conductivity to decrease with increasing frequency or depth for the homogenous model. Another observation based on Table 2, is that the standard deviation of the conductivity distribution in each depth interval tends to decrease with overburden depth for all frequencies. This suggests that deep coal mines have smaller and more evenly distributed conductivity values than shallow coal mines.

#### B. Based on the thin sheet model

It may be possible to explain the behavior of the conductivity on frequency and depth as seen in the previous section by the addition of a thin, highly conducting layer at the surface of the earth [11]. A shallow mine would then have a more weighted contribution from the high conducting surface layer than a deep mine, and the apparent conductivity would decrease with greater depth. The estimated apparent conductivities of those mines with |Q|<0.5 based on the thin sheet model are calculated and sorted into several overburden depth intervals, and then plotted with depth interval as in Fig. 2 for various  $\sigma d$  values and various frequencies. As mentioned earlier, although the conductivity of the thin sheet can be very high, choosing of product value  $\sigma d$  is not arbitrary. The value of  $\gamma_0 d$ , hence the value of d, is required to be small enough to avoid exponential attenuation as the signal travels through the highly conducting thin sheet. One interesting finding is that the apparent conductivity has a greater dependency on the overburden depth at low frequency than at high frequency, as shown in Fig. 2; i.e., the derivative of the exponential curve fit has a greater value at low overburden depth than at greater depths. The exponential curve-fitting equations in these plots provide a good prediction for the depth dependency of the conductivity. This model also predicts the magnitude of decrease in the apparent conductivity with frequency as seen in Table 2, by the appropriate choice of value of  $\sigma d$ .

#### C. Based on Q-factor model

While EM measurements of both transmitting and receiving antennas are needed for the models described above, the Q-factor model or attenuation factor model requires only two parameters: the estimated earth conductivity  $\sigma_0$  and the conductivity thickness product  $\sigma d$  of the thin sheet. Through the use of (15), we can calculate the apparent conductivity. The apparent conductivity values based on this model are plotted in Fig. 3. Again the value of  $\gamma_0 d$ , hence the value of d, is required to be small enough for the reason mentioned in the thin sheet model above. By comparing the exponential coefficients in the fitting functions as shown in Figs 2 and 3, the depth dependency of this model when  $\sigma d = 20$  is very close to that of the thin sheet model. Furthermore, this approach can predict the dependency of apparent conductivity on frequency, as well as on depth, by choosing the value of  $\sigma d$  appropriately.

## D. A regression model based on statistical approach

The conductivity of overburdens above mines in U.S. coal fields can be characterized as a function of overburden depth and operating frequency. The overburdens consist of a large number of horizontal layers of different materials and thicknesses. For any given overburden depth, we can expect overburden characteristics such as conductivity to vary from location to location within the coal fields. Hence, we can develop a statistical approach of sampling a representative number of mines within each of the depth intervals of interest in order to characterize the overburden conductivity, and the corresponding variability about the average, as a function of depth and operating frequency.

In the regression model, overburden apparent conductivity is considered to be related to depth and frequency in an unknown pattern. Up to 94 data points were obtained as a result of field tests conducted at each of four frequency levels. With the assumption that the estimated values of apparent conductivity represent a random sample from a normal distribution with a mean dependent upon both frequency and depth and variance independent of both frequency and depth, a regression model can be obtained to describe the dependency of the apparent conductivity on both frequency and depth as shown in (17). In this model,  $\sigma_a$  is the apparent conductivity to be estimated, and a, b, and c are the regression coefficients to be determined from the mine data. The model values are given in Table 3. The random mine selection process was used to ensure that all measurements can be described by a log-normal probability law. The Cumulative Distribution Function (CDF) of the estimated apparent conductivity (based on the homogenous model) for all frequencies and depths is shown in Fig. 4. From the CDF plot, about 95% of conductivity values fall below 0.5 S/m, and about 60% of them fall below 0.1 S/m. It is worthy to mention that since the values in Table 3 (including standard error) is averaged over all the frequencies and overburden depths (as shown in Table 2), this regression model predicts apparent conductivity more reasonably for low frequencies and shallow mines than for high frequencies and deep mines:

$$\sigma_a = a + b * \log(freq) + c * \log(depth).$$
 (17)

In the regression model as described in (17), the coefficient of frequency is of the same order as that of the depth, so the frequency dependency cannot be ignored in the apparent conductivity estimation in contrast to previous analyses [10].

The apparent conductivity of a specific mine with given overburden depth and operating frequency can then be estimated based on this model, with the related coefficients to fit (17) as shown in Table 3.

#### V. Conclusion

The TTE data from all 94 mines recorded by the BOM in the 1970s were analyzed to estimate the overburden apparent conductivity based on three different models: a homogenous half-space model, a thin sheet model, and a Q-factor model. In the past, full analysis of this data was constrained by computing limitations. The apparent conductivities from the 94 mine data were first estimated based on a homogenous half-space model. The results based on this model show that the apparent conductivity decreases with increasing depth and frequency, which is contrary to the expectations of the model. A thin sheet model was then considered, which is able to provide an explanation for the depth dependency of the conductivity. It also predicts the magnitude of decrease in the apparent conductivity with frequency by appropriately setting the properties of the conducting sheet. Alternately, the Qfactor model was also shown to predict the dependency of apparent conductivity on both frequency and depth by appropriately choosing the properties of a highly conducting thin sheet layer. Among those methods, the thin sheet model provides more reasonable estimation since it considers the effect of the relatively high conducting surface on overall apparent conductivity. By combining the features of the other two models, the Q-factor model also gives a good prediction but is mathematically simple. The conductivity behavior was also described based on a linear-logarithm regression model. The results provided in this paper offer more insight into the overburden apparent conductivity and help to predict the path loss. This estimation of earth conductivity can be used by the mine owner/operator or TTE vendor to predict the performance of the TTE system.

## References

- 1. Durkin, J. U S Bureau of Mines. Pittsburgh Mining and Safety Researcher Center; 1981. Study of through-the-earth transmission data as applied to earth conductivity.
- Hill, DA.; Wait, JR. National Telecommunications & Information Admin. Institute for Telecommunication Sciences; 1982. Theoretical noise and propagation models for through-the-earth communication.
- 3. Wait JR. Mutual electromagnetic coupling of loops over a homogeneous ground. Geophysics. 1955; 20:630–637.
- 4. Wait, JR. Electromagnetic waves in stratified media. The MacMillan Co; 1962.
- 5. Keller, GV.; Frischknecht, FC. Electrical methods in geophysical prospecting. Pergamon; 1966.
- I. IEEE Standard 356. Guide for radio methods of measuring earth conductivity. Institute of Electrical and Electronics Engineers; 1974. p. 18
- 7. Lagace, RL.; Dobbie, JM.; Doerfler, TE.; Hawes, WS.; Spencer, RH. Detection of trapped miner electromagnetic signals above coal mines. Arthur D Little, Inc.; Cambridge, Massachusetts: 1980.
- 8. Wait JR, Spies KP. Evaluation of the surface electromagnetic fields for a buried magnetic dipole source. DTIC Document. 1971

9. Wait JR. Subsurface electromagnetic fields of a circular loop of current located above ground. IEEE Transaction on Antennas and Propagation. 1972; 20:520–522.

- 10. Durkin, J. US Bureau of Mines. Pittsburgh Mining and Safety Research Center; 1984. Apparent earth conductivity over coal mines as estimated from through-the-earth electromagnetic transmission tests.
- 11. Geyer, RG.; Keller, GV.; Ohya, T. Research on the transmission of electromagnetic signals between mine workings and the surface. Colorado School of Mines; 1974.

# **Biographies**



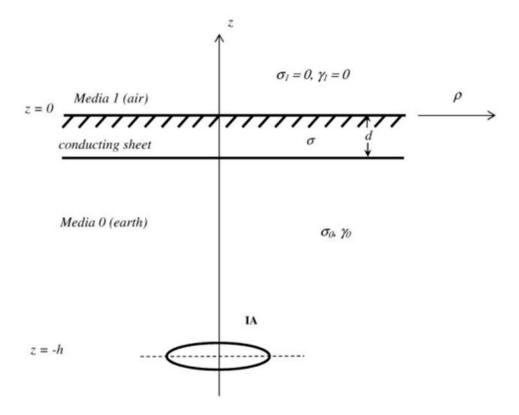
Lincan Yan is a Research Scientist for the National Institute for Occupational Safety and Health (NIOSH). He received his B.S. degree in Mining Engineering from Northeastern University, Shenyang, China; M.E. degree in Engineering Mechanics from Tsinghua University, Beijing, China; M.S. degree in Mechanical Engineering from University of New Mexico (UNM), Albuquerque, NM, USA; and Ph.D. in Electrical Engineering from UNM. Having studied and worked in academia and in the laboratory, he has many years of experience in designing and conducting experiments. He is currently an RF Research Engineer at the National Institute for Occupational Safety and Health, Pittsburgh, PA, USA, where he works on through-the-earth wireless communication projects as a member of the Electrical Safety and Communications Team. By utilizing highly sensitive receive antennas and optimizing transmission, he conducts research on monitoring very weak VLF/ULF radio signals and improving communication range. His work also includes investigating factors that will affect communication in coal mines.



Joseph A. Waynert is a Team Leader at the National Institute for Occupational Safety and Health (NIOSH). His primary research focuses on wireless communications and electronic tracking. In particular, he is experimentally and theoretically investigating the mechanisms controlling path loss in underground coal mining applications. System frequency bands of interest include ELF, MF, and UHF. Prior to NIOSH, Waynert worked in the industry investigating methods of improved spectrum management for the military. Previous to that he worked at Los Alamos National Lab developing applications of applied superconductors. Waynert has a Ph.D. in Physics from the University of Wisconsin-Milwaukee.



Carl Sunderman is a Research Electrical Engineer for the National Institute for Occupational Safety and Health. He has twenty years of experience in the areas of mining equipment automation, geophysical electronics, and radio propagation, and is currently working on projects related to improvements in subterranean communication and pedestrian tracking technologies. He holds a BSEE and MSEE from Gonzaga University.



**Fig. 1.** A small horizontal loop (vertical magnetic dipole) buried in a dissipative half-space with a thin conducting sheet at the surface.

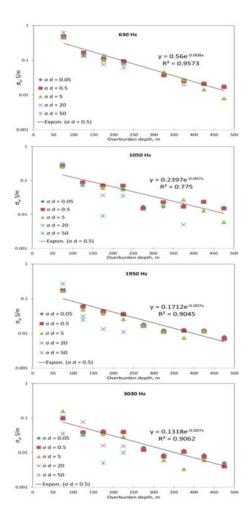


Fig. 2. Estimated apparent conductivities (S/m) change with overburden depth interval (m) for various  $\sigma d$  and frequencies (630 Hz, 1050 Hz, 1950 Hz, 3030 Hz) based on the thin sheet model.

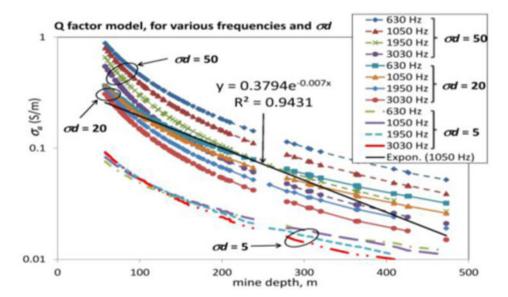
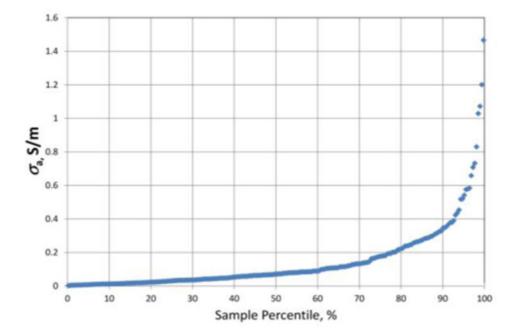


Fig. 3. Apparent conductivity ( $\sigma_a$ , S/m) for different frequencies and  $\sigma d$  based on the Q-factor model.



**Fig. 4.** Estimated apparent conductivities plotted with distribution percentile.

 ${\bf Table~1} \\ {\bf Overburden~depth~distribution~of~94~coal~mines~in~1975}^I \\$ 

Depth m	Depth ft.	Sampling Size	# Of Active Mines
<61.0	<200	2	73
61.3-121.9	201-400	35	369
122.2-182.9	401-600	30	309
183.2-243.8	601-800	13	199
244.1-304.8	800-1000	4	135
305.1-365.8	1001-1200	6	58
>365.8	>1200	4	79

<sup>&</sup>lt;sup>1</sup>MSHA and Bureau of Mines data files as of 1975.

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Table 2

Apparent conductivity ( $\sigma_a$ , S/m) distribution with overburden depth (m) interval for 94 coal mines at different frequencies based on the homogenous half-space earth model; the "std" denotes the conductivity standard deviation in each depth interval

Mine Depth (m)		630 Hz	1050 Hz	1950 Hz	3030 Hz
50 100	Mean	0.6185	0.4202	0.2564	0.1559
20-100	QLS	0.3062	0.3531	0.1767	0.1240
100 150	Mean	09270	0.1301	8580.0	0.0511
001-001	QLS	0.1350	0.0732	0.0465	0.0379
000 051	Mean	0.1425	0.0942	0.0498	0.0452
130-200	QLS	6960.0	0.0548	0.0347	0.0346
036 006	Mean	0.1307	0.0867	0.0378	0.0414
200-230	QLS	9780.0	0.0524	0.0238	0.0233
350 300	Mean	N/A	0.0170	0.0185	0.0140
230-300	STD	N/A	N/A	0.0219	N/A
300 350	Mean	0.0380	0.0250	0.0170	0.0120
500-550	STD	0.0354	0.0156	0.0141	0.0099
350 400	Mean	0.0265	0.0183	0.0130	0.0118
550-400	STD	0.0261	0.0153	0.0076	0.0049
400 450	Mean	0.0220	0.0245	0.0230	0.0000
400-450	STD	0.0118	0.0035	N/A	0.0085
005 051	Mean	0.0180	0.0170	0.0080	0.0050
450-500	STD	N/A	N/A	N/A	N/A

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Table 3 Regression model for apparent conductivity ( $\sigma_a$ , S/m) with respect to log depth (m) and log frequency (Hz)

Observations	238
а	2.1834
b	-0.2932
с	-0.5068
Standard Error	0.1479
R Square	0.4674